Int J Thermophys (2010) 31:2337–2347
DOI 10.1007/s10765-010-0881-y

Quasi-static Thermal Deflection of a Thin Clamped Hollow Circular Disk Due to Heat Generation

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Received: 18 September 2009 / Accepted: 9 November 2010 / Published online: 1 December 2010
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Abstract This paper deals with the determination of the thermal deflection in a thin clamped hollow circular disk defined as $a \le r \le b$; $0 \le z \le h$ under an unsteady temperature field due to internal heat generation within it. A thin hollow circular disk is considered having an arbitrary initial temperature and subjected to heat flux at the outer circular boundary ($r = b$) where an inner circular boundary ($r = a$) is at zero heat flux. Also, the upper surface ($z = h$) and the lower surface ($z = 0$) of the disk are at zero temperature. The governing heat conduction equation has been solved by using an integral transform technique. The inner and outer edges of the disk are clamped $\frac{\partial \omega}{\partial r} = 0$ at $r = a, r = b$. The results are obtained in a series form in terms of Bessel's functions and are illustrated graphically.

Keywords Heat generation · Non-homogeneous heat conduction equation · Thermal deflection · Thermoelastic problem

1 Introduction

Boley and Weiner [1] studied the problems of thermal deflection of an axisymmetric heated circular plate in the case of fixed and simply supported edges. Roy Choudhuri

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[2] discussed the normal deflection of a thin clamped circular plate due to ramp-type heating of a concentric circular region of the upper face. This satisfies the time-dependent heat conduction equation. Deshmukh and Khobragade [3] has determined a quasi-static thermal deflection in a thin circular plate due to partially distributed and axisymmetric heat supply on the outer curved surface with the upper and lower faces at zero temperature. Deshmukh et al. [4] has determined the thermal stresses in a hollow circular disk due to internal heat generation within it. Recently, Deshmukh et al. [5] studied the thermal deflection in a thin circular plate subjected to heat generation within it.

In this paper the work of Deshmukh et al. [5] has been extended for a two-dimensional non-homogeneous boundary value problem of heat conduction, and the thermal deflection of thin clamped hollow circular disk defined as $a \leq r \leq b$; $0 \leq z \leq h$ due to internal heat generation within it has been studied. A thin hollow circular disk is considered having an arbitrary initial temperature and subjected to heat flux at the outer circular boundary $(r = b)$ where an inner circular boundary $(r = a)$ is at zero heat flux. Also, the upper surface $(z = h)$ and the lower surface $(z = 0)$ of the disk are at zero temperature. The governing heat conduction equation has been solved by using an integral transform technique. The results are obtained in series form in terms of Bessel's functions. The results for the thermal deflection have been computed numerically and are illustrated graphically. It is believed that this particular problem has not been previously considered.

The rotating disk has applications in aerospace engineering, particularly in gas turbines and gears. The rotating disk represents work under thermo-mechanical loads.

2 Formulation of the Problem

Consider a thin hollow circular disk of thickness h occupying space D defined by $a \leq r \leq b, 0 \leq z \leq h$. Initially, the disk is kept at an arbitrary temperature $F(r, z)$. The inner circular boundary $(r = a)$ is at zero temperature, whereas the heat flux $\frac{Q(z,t)}{k}$ is applied on the outer circular boundary $(r = b)$. Also, the upper surface $(z = h)$and the lower surface $(z = 0)$ of the disk are at zero temperature. For time $t > 0$, heat is generated within the thin hollow circular disk at the rate $g(r, z, t)$. Under these conditions, the thermal deflection in a thin hollow circular disk due to heat generation is required to be determined (see Fig. 1).

2.1 Governing Equations of Thermal Deflection

The differential equation satisfying the deflection function $\omega(r, t)$ is given as

$$\nabla^4 \omega = -\frac{\nabla^2 M_\mathrm{T}}{D(1-\nu)} \tag{1}$$

where M_T is the thermal moment of the disk defined as

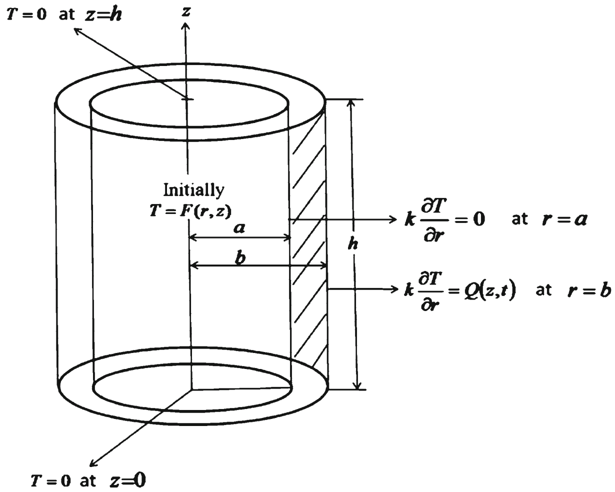

Fig. 1 Geometry of the heat conduction problem

$$M_{\mathrm{T}} = a_t E \int_0^h T(r, z, t) z \mathrm{d}z \tag{2}$$

D is the flexural rigidity of the disk denoted as

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{3}$$

a_t, E, and ν are the coefficients of the linear thermal expansion, the Young's modulus, and Poisson's ratio of the disk material, respectively, and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \tag{4}$$

Since, the inner and outer edges of the hollow circular disk are clamped,

$$\frac{\partial \omega}{\partial r} = 0 \quad \text{at} \quad r = a \quad \text{and} \quad r = b \tag{5}$$

Initially, $T = \omega = F(r, z)$ at $t = 0$.

2.2 Governing Heat Conduction Equation

The temperature of the hollow circular disk $T(r, z, t)$ at time t satisfies the differential equation,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{g(r, z, t)}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{in} \quad a \le r \le b,\ 0 \le z \le h \tag{6}$$

with the boundary conditions,

$$k\frac{\partial T}{\partial r} = 0 \quad \text{at} \quad r = a,\ t > 0 \tag{7}$$

$$k\frac{\partial T}{\partial r} = Q(z,t) \quad \text{at} \quad r = b,\ t > 0 \tag{8}$$

$$T = 0 \quad \text{at} \quad z = 0,\ t > 0 \tag{9}$$

$$T = 0 \quad \text{at} \quad z = h,\ t > 0 \tag{10}$$

and initial condition,

$$T(r,z,t) = F(r,z), \quad \text{in} \quad a \le r \le b,\ 0 \le z \le h \quad \text{for} \quad t = 0 \tag{11}$$

where K and α are the thermal conductivity and thermal diffusivity, respectively, of the material of the hollow circular disk.

Equations 1–11 constitute a mathematical formulation of the problem.

3 Solution

3.1 Determination of Temperature, T

To obtain an expression for the temperature $T(r,z,t)$, following Ozisik [6], we develop the finite Fourier transform and finite Hankel transform and their respective inverses and operate them on Eqs. 6–11;

$$\begin{aligned} T(r,z,t) = \sum_{p=1}^{\infty}\sum_{m=1}^{\infty} & K(\eta_p,z)K_0(\beta_m,r)\mathrm{e}^{-\alpha(\beta_m^2+\eta_p^2)t} \\ & \times \left\{ \int_{r'=a}^{b}\int_{z'=0}^{h} r'K_0(\beta_m,r')K(\eta_p,z')F(r',z')\mathrm{d}r'\mathrm{d}z' \right. \\ & + \int_{t'=0}^{t} \mathrm{e}^{\alpha\left(\beta_m^2+\eta_p^2\right)t'} \left(\frac{\alpha}{K}\int_{r'=a}^{b}\int_{z'=0}^{h} r'K_0(\beta_m,r')K(\eta_p,z') \right. \\ & \left.\left. \times\, g(r',z',t')\mathrm{d}r'\mathrm{d}z' + \frac{\alpha}{K}bK_0(\beta_m,b)\int_{z'=0}^{h} K(\eta_p,z')Q(z',t')\mathrm{d}z' \right)\mathrm{d}t' \right\} \end{aligned} \tag{12}$$

where

$$K(\eta_p,z) = \sqrt{\frac{2}{h}}\sin(\eta_p z) \tag{13}$$

and $\eta_1, \eta_2, \ldots$ are the positive roots of the transcendental equation

$$\sin(\eta_p h) = 0, \;\; p = 1, 2, \ldots \tag{14}$$

$$K_0(\beta_m, r) = \frac{\pi}{\sqrt{2}} \frac{\beta_m \cdot J_0'(\beta_m b) \cdot Y_0'(\beta_m b)}{\left[1 - \frac{J_0'^2(\beta_m b)}{J_0'^2(\beta_m a)}\right]^{\frac{1}{2}}} \left[\frac{J_0(\beta_m r)}{J_0'(\beta_m b)} - \frac{Y_0(\beta_m r)}{Y_0'(\beta_m b)}\right] \tag{15}$$

and $\beta_1, \beta_2, \ldots$ are the positive roots of the transcendental equation

$$\frac{J_0'(\beta a)}{J_0'(\beta b)} - \frac{Y_0'(\beta a)}{Y_0'(\beta b)} = 0. \tag{16}$$

3.2 Determination of Thermal Deflection $\omega(r, t)$

Using Eq. 12 in Eq. 2, one obtains

$$\begin{aligned} M_{\mathrm{T}} = & -\sqrt{\frac{2}{h}} a_t E h \sum_{p=1}^{\infty} \sum_{m=1}^{\infty} \frac{\cos(\eta_p h)}{\eta_p} K_0\,(\beta_m, r) \mathrm{e}^{-\alpha(\beta_m^2 + \eta_p^2)t} \\ & \times \left\{ \int_{r'=a}^{b} \int_{z'=0}^{h} r' K_0(\beta_m, r') K(\eta_p, z') F(r', z') \mathrm{d}r' \mathrm{d}z' \right. \\ & + \int_{t'=0}^{t} \mathrm{e}^{\alpha(\beta_m^2 + \eta_p^2)t'} \left(\frac{\alpha}{K} \int_{r'=a}^{b} \int_{z'=0}^{h} r' K_0(\beta_m, r') K(\eta_p, z') g(r', z', t') \mathrm{d}r' \mathrm{d}z' \right. \\ & \left. \left. + \; \frac{\alpha}{K} b K_0(\beta_m, b) \int_{z'=0}^{h} K(\eta_p, z') Q\left(z', t'\right) \mathrm{d}z' \right) \mathrm{d}t' \right\} \end{aligned} \tag{17}$$

Assume the solution of Eq. 1 satisfies the condition of Eq. 5 as

$$\omega(r, t) = \sum_{m=1}^{\infty} C_m(t) \left[\frac{J_0(\beta_m r)}{J_0'(\beta_m b)} - \frac{Y_0(\beta_m r)}{Y_0'(\beta_m b)}\right] \tag{18}$$

where β_m' are the positive roots of the transcendental equation,

$$\frac{J_0'(\beta a)}{J_0'(\beta b)} - \frac{Y_0'(\beta a)}{Y_0'(\beta b)} = 0.$$

It can be easily shown that

$$\frac{\partial \omega}{\partial r} = \sum_{m=1}^{\infty} C_m(t)\left[\frac{J_0'(\beta_m r)}{J_0'(\beta_m b)} - \frac{Y_0'(\beta_m r)}{Y_0'(\beta_m b)}\right]$$
$$\frac{\partial \omega}{\partial r} = 0 \quad \text{at} \quad r = a$$

Now,

$$\frac{\partial \omega}{\partial r} = \sum_{m=1}^{\infty} C_m(t)\left[\frac{J_0'(\beta_m r)}{J_0'(\beta_m b)} - \frac{Y_0'(\beta_m r)}{Y_0'(\beta_m b)}\right]$$
$$\frac{\partial \omega}{\partial r} = 0 \quad \text{at} \quad r = b.$$

Hence, the solution of Eq. 18 satisfies the condition of Eq. 5.

Now,

$$\nabla^4 \omega = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^2 \sum_{m=1}^{\infty} C_m(t)\left[\frac{J_0(\beta_m r)}{J_0'(\beta_m b)} - \frac{Y_0(\beta_m r)}{Y_0'(\beta_m b)}\right] \tag{19}$$

Using the well known results,

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right) J_0(\beta_m r) = -\beta_m^2 J_0(\beta_m r)$$
$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right) Y_0(\beta_m r) = -\beta_m^2 Y_0(\beta_m r)$$

in Eq. 19, one obtains

$$\nabla^4 \omega = \sum_{m=1}^{\infty} C_m(t)\beta_m^4 \left[\frac{J_0(\beta_m r)}{J_0'(\beta_m b)} - \frac{Y_0(\beta_m r)}{Y_0'(\beta_m b)}\right] \tag{20}$$

Also,

$$\nabla^2 M_{\mathrm{T}} = -\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\sqrt{\frac{2}{h}} a_t E h \sum_{p=1}^{\infty}\sum_{m=1}^{\infty} \frac{1}{\eta_p} \cos(\eta_p h) K_0\,(\beta_m, r) \mathrm{e}^{-\alpha(\beta_m^2+\eta_p^2)t}$$
$$\times \left\{ \int_{r'=a}^{b} \int_{z'=0}^{h} r' K_0(\beta_m, r') K(\eta_p, z') F(r', z') \mathrm{d}r' \mathrm{d}z' \right.$$
$$+ \int_{t'=0}^{t} \mathrm{e}^{\alpha(\beta_m^2+\eta_p^2)t'} \left(\frac{\alpha}{K} \int_{r'=a}^{b} \int_{z'=0}^{h} r' K_0(\beta_m, r') K(\eta_p, z') g(r', z', t') \mathrm{d}r' \mathrm{d}z' \right.$$

$$+ \frac{\alpha}{K} b K_0(\beta_m, b) \int_{z'=0}^{h} K(\eta_p, z') Q(z', t') \mathrm{d}z' \Bigg) \mathrm{d}t' \Bigg\}$$

$$\nabla^2 M_{\mathrm{T}} = \sqrt{\frac{2}{h}} a_t E h \sum_{p=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{\eta_p} \cos(\eta_p h) \beta_m^2 K_0(\beta_m, r) \mathrm{e}^{-\alpha(\beta_m^2+\eta_p^2)t}$$
$$\Bigg\{ \int_{r'=a}^{b} \int_{z'=0}^{h} r' K_0(\beta_m, r') K(\eta_p, z') F(r', z') \mathrm{d}r' \mathrm{d}z'$$
$$+ \int_{t'=0}^{t} \mathrm{e}^{\alpha(\beta_m^2+\eta_p^2)t'} \Bigg(\frac{\alpha}{K} \int_{r'=a}^{b} \int_{z'=0}^{h} r' K_0(\beta_m, r') K(\eta_p, z') g(r', z', t') \mathrm{d}r' \mathrm{d}z'$$
$$+ \frac{\alpha}{K} b K_0(\beta_m, b) \int_{z'=0}^{h} K(\eta_p, z') Q\left(z', t'\right) \mathrm{d}z' \Bigg) \mathrm{d}t' \Bigg\} \tag{21}$$

Substituting Eqs. 20 and 21 into Eq. 1, one obtains

$$\sum_{m=1}^{\infty} C_m(t) \beta_m^4 \left[\frac{J_0(\beta_m r)}{J_0'(\beta_m b)} - \frac{Y_0(\beta_m r)}{Y_0'(\beta_m b)} \right]$$
$$= -\sqrt{\frac{2}{h}} \frac{a_t E}{D(1-v)} h \sum_{p=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{\eta_p} \cos(\eta_p h) \beta_m^2 \frac{\pi}{\sqrt{2}}$$
$$\times \frac{\beta_m \cdot J_0'(\beta_m b) \cdot Y_0'(\beta_m b)}{\left[1 - \frac{J_0'^2(\beta_m b)}{J_0'^2(\beta_m a)} \right]^{\frac{1}{2}}} \left[\frac{J_0(\beta_m r)}{J_0'(\beta_m b)} - \frac{Y_0(\beta_m r)}{Y_0'(\beta_m b)} \right]$$
$$\times \mathrm{e}^{-\alpha(\beta_m^2+\eta_p^2)t} \Bigg\{ \int_{r'=a}^{b} \int_{z'=0}^{h} r' K_0(\beta_m, r') K(\eta_p, z') F(r', z') \mathrm{d}r' \mathrm{d}z'$$
$$+ \int_{t'=0}^{t} \mathrm{e}^{\alpha(\beta_m^2+\eta_p^2)t'} \Bigg(\frac{\alpha}{K} \int_{r'=a}^{b} \int_{z'=0}^{h} r' K_0(\beta_m, r') K(\eta_p, z') g(r', z', t') \mathrm{d}r' \mathrm{d}z'$$
$$+ \frac{\alpha}{K} b K_0(\beta_m, b) \int_{z'=0}^{h} K(\eta_p, z') Q\left(z', t'\right) \mathrm{d}z' \Bigg) \mathrm{d}t' \Bigg\} \tag{22}$$

Solving Eq. 22, one obtains

$$C_m(t) = -\sqrt{\frac{2}{h}}\frac{a_t E h}{D(1-v)}\sum_{p=1}^{\infty}\frac{1}{\eta_p}\cos(\eta_p h)\frac{1}{\beta_m^2}\frac{\pi}{\sqrt{2}}\frac{\beta_m \cdot J_0'(\beta_m b)\cdot Y_0'(\beta_m b)}{\left[1-\frac{J_0'^2(\beta_m b)}{J_0'^2(\beta_m a)}\right]^{\frac{1}{2}}}$$
$$\times\, \mathrm{e}^{-\alpha\left(\beta_m^2+\eta_p^2\right)t}\left\{\int\limits_{r'=a}^{b}\int\limits_{z'=0}^{h} r' K_0(\beta_m, r')K(\eta_p, z')F(r', z')\mathrm{d}r'\mathrm{d}z'\right.$$
$$+\int\limits_{t'=0}^{t}\mathrm{e}^{\alpha(\beta_m^2+\eta_p^2)t'}\left(\frac{\alpha}{K}\int\limits_{r'=a}^{b}\int\limits_{z'=0}^{h} r' K_0(\beta_m, r')K(\eta_p, z')g(r', z', t')\mathrm{d}r'\mathrm{d}z'\right.$$
$$\left.\left.+\frac{\alpha}{K}bK_0(\beta_m, b)\int\limits_{z'=0}^{h}K(\eta_p, z')Q(z', t')\mathrm{d}z'\right)\mathrm{d}t'\right\} \tag{23}$$

Substituting Eq. 23 in Eq. 18, one obtains

$$\omega(r,t) = -\sqrt{\frac{2}{h}}\frac{a_t E h}{D(1-v)}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\eta_p}\cos(\eta_p h)\frac{1}{\beta_m}\frac{\pi}{\sqrt{2}}\frac{J_0'(\beta_m b)Y_0'(\beta_m b)}{\left[1-\frac{J_0'^2(\beta_m b)}{J_0'^2(\beta_m a)}\right]^{\frac{1}{2}}}$$
$$\times\left[\frac{J_0(\beta_m r)}{J_0'(\beta_m b)}-\frac{Y_0(\beta_m r)}{Y_0'(\beta_m b)}\right]$$
$$\times\left[\mathrm{e}^{-\alpha(\beta_m^2+\eta_p^2)t}\left\{\int\limits_{r'=a}^{b}\int\limits_{z'=0}^{h} r' K_0(\beta_m, r')K(\eta_p, z')F(r', z')\mathrm{d}r'\mathrm{d}z'\right.\right.$$
$$+\int\limits_{t'=0}^{t}\mathrm{e}^{\alpha(\beta_m^2+\eta_p^2)t'}\left(\frac{\alpha}{K}\int\limits_{r'=a}^{b}\int\limits_{z'=0}^{h} r' K_0(\beta_m, r')K(\eta_p, z')g(r', z', t')\mathrm{d}r'\mathrm{d}z'\right.$$
$$\left.\left.\left.+\frac{\alpha}{K}bK_0(\beta_m, b)\int\limits_{z'=0}^{h}K(\eta_p, z')Q(z', t')\mathrm{d}z'\right)\mathrm{d}t'\right\}\right] \tag{24}$$

4 Special Case and Numerical Calculations

$$\begin{aligned}\text{Setting: } F(r,z) &= (r^2-a^2)^2 \times r^2 \times z \times (z-h)\\ Q(z,t) &= z \times (z-h) \times \mathrm{e}^{-\varphi t}\\ g(r,z,t) &= g_{\mathrm{pi}}\delta(r-r_1)\delta(z-z_1)\delta(t-\tau)\end{aligned}$$

where r is the radius measured in m, δ is the Dirac-delta function, and $\phi = 5\ \mathrm{s}^{-1} > 0$.

The heat source $g(r,z,t)$ is an instantaneous point heat source of strength $g_{\mathrm{pi}} = 50\,\mathrm{J} \cdot \mathrm{m}^{-1}$ situated at the center of the hollow circular disk along the radial direction and axial direction and releases its heat spontaneously at time $t \to \tau = 5$ s.

4.1 Dimensions

Inner radius of a thin hollow circular disk: $a = 1\,\mathrm{m}$
Outer radius of a thin hollow disk: $b = 2\,\mathrm{m}$
Thickness of hollow disk: $h = 0.2\,\mathrm{m}$
Central circular path of disk in radial and axial directions: $r_1 = 1.5\,\mathrm{m}$ and $z_1 = 0.1\,\mathrm{m}$

4.2 Material Properties

The numerical calculation has been carried out for a copper (pure) thin hollow circular disk with the following material properties:

Thermal diffusivity $\alpha = 112.34 \times 10^{-6}\,\mathrm{m}^2 \cdot \mathrm{s}^{-1}$
Thermal conductivity $k = 386\,\mathrm{W} \cdot \mathrm{m}^{-1} \cdot \mathrm{K}^{-1}$
Density $\rho = 8954\,\mathrm{kg} \cdot \mathrm{m}^{-3}$
Specific heat $c_p = 383\,\mathrm{J} \cdot \mathrm{kg}^{-1} \cdot \mathrm{K}^{-1}$
Poisson ratio $\nu = 0.35$
Coefficient of linear thermal expansion, $a_t = 16.5 \times 10^{-6}\,\mathrm{K}^{-1}$
Lamé constant $\mu = 26.67$

4.3 Roots of the Transcendental Equation

Let $\beta_1 = 3.1965$, $\beta_2 = 6.3123$, $\beta_3 = 9.4445$, $\beta_4 = 12.5812$, and $\beta_5 = 15.7199$ are the positive roots of transcendental equation $\left(\frac{J_0'(\beta_m)}{J_0'(2\beta_m)} - \frac{Y_0'(\beta_m)}{Y_0'(2\beta_m)}\right) = 0$.

The numerical calculation has been carried out for $z = 0.15$ m and $t = 5$ s with the help of computational mathematical software Mathcad-2000 and the graphs are plotted with the help of Excel (MS office-2007).

For convenience we set $X = \frac{a_t E h}{D(1-\nu)}$, i.e., elastic material constants.

5 Discussion of the Results

In this paper we extended the work of Deshmukh et al. [5] in a two-dimensional inhomogeneous boundary value problem of heat conduction and determined the expressions for temperature and deflection due to internal heat generation within it.

As a special case, a mathematical model is constructed for copper (pure), with the thin clamped hollow circular disk with the material properties specified as above. The heat source is an instantaneous point heat source of strength g_{pi} situated at the center of the circular plate along the radial and axial directions and releases its heat instantaneously at the time $t \to \tau$.

In Fig. 2, it can be observed that the temperature is fluctuating in the regions $1 \le r \le 1.2$, $1.2 \le r \le 1.6$, $1.6 \le r \le 1.9$. With the heat flows towards the outer circular edge of the hollow circular disk, the clamped support will not allow this flow, so heat flows move towards the lower surface in axial directions.

In Fig. 3, it can be observed that the deflection is a maximum at the inner circular edge and decreases from the inner-to-outer circular edge.

We can surmise that due to internal heat generation within a thin hollow circular disk, a deflection occurs at the inner circular edge. The direction of heat flow and direction of body deflection are the same, and they are proportionate.

On comparing this result with the result discussed in [2], we observed that the deflection rate increases due to internal heat generation in a hollow circular disk.

The results presented here will be useful in engineering problems particularly in aerospace engineering for stations of a missile body not influenced by nose tapering. The missile skill material is assumed to have physical properties independent of tem-

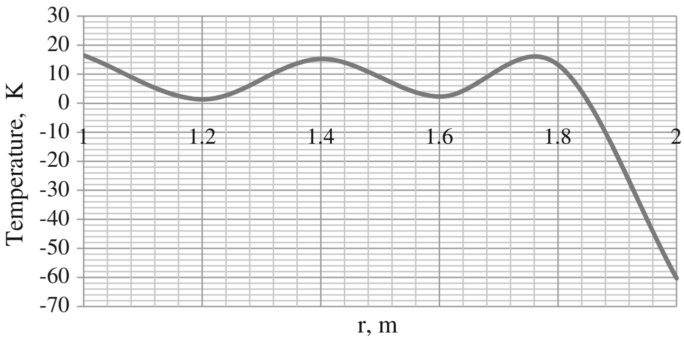

Fig. 2 Temperature distribution

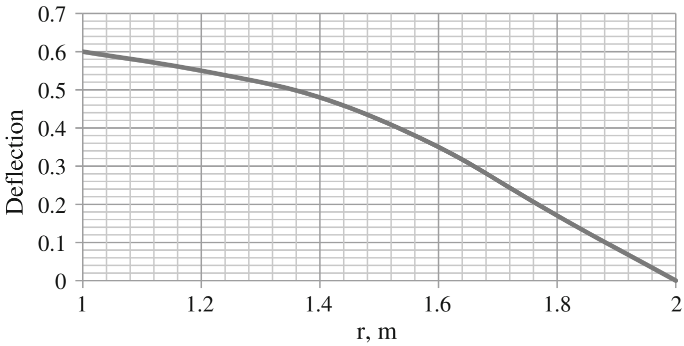

Fig. 3 Thermal deflection

perature, so that the temperature $T(r, z, t)$ is a function of the radius, thickness, and time only.

Also, any particular case of special interest can be derived by assigning suitable values to the parameters and functions in Eqs. 12 and 24.

Acknowledgments The authors express sincere thanks to reviewers for their valuable suggestions. Also the authors are thankful to University Grants Commission, New Delhi, India for providing partial financial assistance under a major research project scheme.

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